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ORIGINAL ARTICLE

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Diversity control in ant colony optimization

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Abstract Optimization inspired by cooperative food retrieval in ants has been unexpectedly successful and has been known as ant colony optimization (ACO) in recent years. One of the most important factors to improve the performance of the ACO algorithms is the complex trade-off between intensification and diversification. This article investigates the effects of controlling the diversity by adopting a simple mechanism for random selection in ACO. The results of computer experiments have shown that it can generate better solutions stably for the traveling salesmen problem than AS_{rank} which is known as one of the newest and best ACO algorithms by utilizing two types of diversity.

Key words Ant colony optimization · Diversity · Diversification · Intensification · Pheromone communication

1 Introduction

Ant colony optimization (ACO)[1,2] is a new search metaphor for solving combinatorial optimization problems, and has been unexpectedly successful in recent years. It has been applied to many combinatorial optimization problems, e.g., traveling salesman problems (TSP), network routing problems, graph coloring problems, quadratic assignment problems, and others. ACO is currently the best available meta-heuristic for some problems, and is among the most competitive approaches for the other problems,[3,4] while several such heuristic algorithms have been proposed which include algorithms like simulated annealing, tabu search, genetic algorithms, and artificial neural network.

In general, one of the most important themes in the study of heuristic algorithms is the balance between intensification and diversification. Too much emphasis on the former can make agents converge to a local optimum and too much emphasis on the latter can cause an unstable state, although these two factors are essential because we need the former to accelerate convergence and the latter to find better solutions.

This article investigates the effects of explicitly controlling the diversity so as to adjust the trade-off between intensification and diversification by adopting a simple mechanism for random selection in ACO. Sect. 2 explains the ACO algorithms. Sect. 3 briefly surveys the ACO research in terms of the balance between intensification and diversification, and describes our idea on diversity control in ACO. Sect. 4 reports the results of the computer experiments applied to the traveling salesman problem, and analyzes the interaction between two types of diversity underlying in the ACO framework based on the results. Sect. 5 summarizes the paper.

2 Ant colony optimization

Recently, computer scientists have been able to transform models of social insect collective behavior into useful optimization and control algorithms. A new line of research focuses on the transformation of knowledge from how social insects collectively solve problems into artificial problem-solving techniques, producing a form of artificial intelligence, or swarm intellingence.[3,5] In particular, optimization algorithms inspired by models of cooperative food retrieval in ants have been unexpectedly successful and have become known as ACO.

The essential framework of the ACO is to parallelize searches over several constructive computational threads, based on a memory structure incorporating the information about the effectiveness of previously obtained fragments

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of solutions. This structure is maintained dynamically by deposition, evaporation, and detection of conceptual pheromone. These processes realize pheromone-based indirect and asynchronous communication among ants mediated by an environment, and form a positive feedback where effective fragments of solutions will receive a greater amount of pheromone and in turn a larger number of ants will choose the fragments of solutions and deposit pheromone.

The first ACO algorithm, called Ant System (AS), was initially proposed by Dorigo et al.[6] and applied to the well-known traveling salesman problem (TSP) as a benchmark problem. As has been the prototype of many following ACO algorithms with which many other combinatorial problems can be successfully solved.

In TSP a given set of n cities has to be traversed so that every city is visited exactly once and the tour ends in the initial city. The optimization goal is to find a shortest possible tour. Let d_{ij} be the distance between city i and city j and τ_{ij} the amount of pheromone in the edge that connects i and j.

Each of m ants decides independently on the city to be visited next based on the intensity of the pheromone trail τ_{ij} and a heuristic value η_{ij}, until the tour is completed, where the heuristic value $\eta_{ij} = 1/d_{ij}$ is generally used. Each ant maintains a list N_i^k of cities that remain to be visited. An ant located at city i selects an edge between city i and city j according to the probability:

$$p_{ij}^k(t) = \frac{[\tau_{ij}(t)]^\alpha (\eta_{ij})^\beta}{\sum_{l \in N_i^k} [\tau_{il}(t)]^\alpha (\eta_{il})^\beta} \qquad \forall j \in N_i^k, \tag{1}$$

where the parameters α and β determine the relative influence of pheromone and distance, respectively. After every ant completes a tour, pheromone is deposited:

$$\Delta\tau_{ij}^k(t) = \begin{cases} Q/L^k(t) & \text{if } (i,j) \in T^k(t), \\ 0 & \text{if } (i,j) \notin T^k(t), \end{cases} \tag{2}$$

where Q is a constant, $L^k(t)$ is the length of the tour generated by the ant k at iteration t and $T^k(t)$ is the set of edges constituting it. In this manner, the shorter a tour that an edge constitutes, the more pheromone is laid on the edge. The amount of pheromone is updated according to the rule:

$$\tau_{ij}(t+1) = \rho\tau_{ij}(t) + \sum_{k=1}^{m} \Delta\tau_{ij}^k(t), \tag{3}$$

where ρ $(0 < \rho < 1)$ represents the persistence of pheromone trails ($1 - \rho$ means the evaporation rate). These processes are repeated a predefined number of times $t_{\max}$.

3 Diversity control

Each ant constructs a tour by explicitly using elements of previous effective solutions in ACO. The underlying idea of this process is parallel with the concept of the building block hypothesis in genetic algorithms, while ACO not only identifies building blocks but also identifies correlations between building blocks.[3]

One of the central themes of the research of this type of meta-heuristics including evolutionary computation is the balance between intensification (exploitation of the previous solutions) and diversification (exploration of the search space). In genetic algorithm (GA), fitness evaluation and selection are directly related to intensification, mutation is directly related to diversification, and crossover concerns both of them. The trade-off between them can be explicitly controlled by adjusting the mutation rate. In ACO, pheromone depositing is equivalent to fitness evaluation and edge selection by ants is comparable to selection and crossover in GA.

Here we briefly survey the research that has been conducted to extend or modify the first algorithm ant system (AS) in terms of the balance between exploitation and exploration.

The ant colony system (ACS) develop by Dorigo and Gambardella[4] adopts a modified selection rule called the pseudo random proportional rule which favors transitions toward nodes connected by short edges and with a large amount of pheromone. ACS also differs from AS in that the updating rule is applied only to edges belonging to the global best tour (the shortest tour from the beginning of the trial). These two modifications accelerate intensification.

Stützle and Hoos[7] proposed the MAX–MIN ant system (MMAS), an extension of AS applied to TSP. Possible trial values are restricted to the interval $[\tau_{\min}, \tau_{\max}]$, where these two parameters are set up in a problem-dependent way. This modification prevents ants from converging to a local optimum. MMAS also adopts a concept of elitism in which only the best ant at each iteration updates trails. These are the main differences between MMAS and AS, aiming to achieve good balance between exploitation and exploration. Moreover, Stützle and Hoos proposed an additional mechanism called pheromone trail smoothing (PTS). This mechanism helps to achieve a more efficient exploration.

Bullnheimer et al.[8] proposed another modification of AS, called AS_{rank}, introducing a rank-based version of the probability distribution which allows only $\sigma - 1$ "elite ants" to deposit pheromone based on tour length. At the same time, the best tour found so far is also updated in consideration of exploitation. The updating rule is described as follows:

$$\tau_{ij}(t+1) = \rho\tau_{ij}(t) + \sum_{\mu=1}^{\sigma-1} \Delta\tau_{ij}^\mu(t) + \Delta\tau_{ij}^*(t), \tag{4}$$

$$\Delta\tau_{ij}^\mu(t) = \begin{cases} (\sigma-\mu)Q/L^\mu(t) & \text{if } (i,j) \in T^\mu(t), \\ 0 & \text{if } (i,j) \notin T^\mu(t), \end{cases} \tag{5}$$

$$\Delta\tau_{ij}^*(t) = \begin{cases} \sigma Q/L^\mu(t) & \text{if } (i,j) \in T^*(t), \\ 0 & \text{if } (i,j) \notin T^*(t), \end{cases} \tag{6}$$

where $L^\mu(t)$ is the length of the tour generated by the μ-th best ant at iteration t, $T^\mu(t)$ is the set of edges constituting it,

$L^*(t)$ is the length of the tour generated by the best ant by iteration t, and $T^*(t)$ is the set of edges constituting it.

AS_{elite} was also proposed by Bullnheimer et al. as the first extension of AS prior to AS_{rank}. The idea is only to give extra emphasis to the best tour found so far without ranking by adding Eq. 6 to the updating rule in AS as follows:

$$\tau_{ij}(t+1) = \rho\tau_{ij}(t) + \sum_{k=1}^{m} \Delta\tau_{ij}^{k} + \Delta\tau_{ij}^{*}. \quad (7)$$

In general, there are supposed to be at least two types of diversity in ACO: (a) diversity in finding tours, and (b) diversity in depositing pheromone. We examine the effects of explicitly controlling diversity (a), while most of the ACO algorithms proposed so far have placed emphasis on adjusting diversity (b). We believe that control of diversity (a) would work better because the effect of control of diversity (a) is adjusted automatically based on adopted mechanisms that realize some sort of elitism, whereas control of diversity (b) directly affects diversity (a) in general.[9]

For this purpose we introduce in ACO a mechanism of random selection in addition to the original selection defined by Eq. 1 that is base on both pheromone trails and heuristic information. Random selection adopted in this study is a very simple operation which selects a city among unvisited cities with equal probability, whose function is equivalent to the mutation adopted in evolutionary computation. Random selection rate r is a probability with which random selection is operated every time each ant selects the next city, and this parameter adjusts the balance between exploitation and exploration continuously. Two schemes for finding better tours exist, and are based on introduced random selection.

In the first scheme, increase in diversity in finding tours by random selection directly results in finding of the global best tours. In other words, tours containing an edge or edges taken by random selection update the global best tours. Figure 1a shows a simple example of finding a global best tour. Pheromone trails are represented by shaded lines, cities are represented by circles, and tours are represented by solid lines in this figures. An ant may start from city 1 and move to city 2. In the case that an ant selects the edge between city 6 and city 8 by random selection, the tour obtained is shorter than the tour generated by following the pheromone trail.

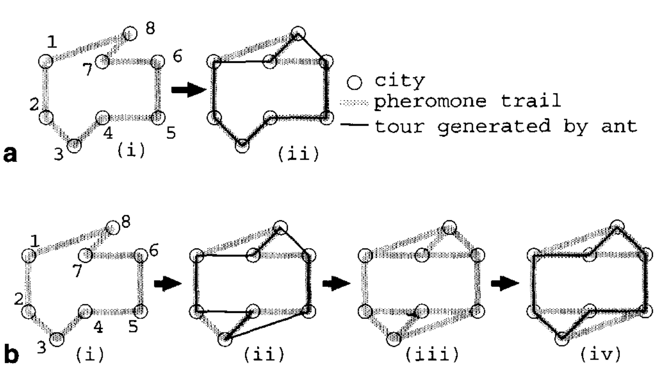

Fig. 1. Two simple examples of searching for a best tour **a** by controlling diversity, and **b** by controlling two types of diversity

In the second scheme, increase in diversity in finding tours by random selection affects diversity in depositing pheromone in the first stage and this enables a new global best tour to be found in the second stage. Two types of diversity underlying in ACO are cleverly used in this scheme. Consider the simple scenario shown in Fig. 1b as an example. There is a pheromone trail (i). Then an ant using random selection finds another tour (ii). The ant deposits pheromone (iii). Finally another ant in the following generations finds the global best tour without random selection (iv).

Thought it can be introduced in any ACO algorithms in principle, this article examines the case that the mechanism of random selection is introduced in AS_{rank} for the following reasons: (1) tours containing an edge or edges taken by random selection are supposed to be worse because random selection disregards not only pheromone information but also heuristic information. Therefore, it becomes essential to adopt the idea of elite ants like the ones in AS_{rank} because the effect of diversity control on the pheromone diversity is adjusted implicitly; (2) AS_{rank} sometimes showed a tendency of performance limit caused by the emphasis on intensification in our preliminary experiments; (3) AS_{rank} is one of the newest versions of ACO and is reported to outperform the other ACO methods.[8]

4 Experiments

4.1 Setting

We analyzed the effects of diversity control by introducing random selection in AS_{rank}, based on computational experiments. We tested the extended algorithm on eli51.tsp including 51 cities from the TSPLIB (www.iwr.uni-heidelberg.de/groups/comopt/software/TSPLIB95), where optimal tour lengths are 426. Random selection rate r was set at [0.01, 0.1]. The number of ants was set to the number of the cities. ACO parameters were set to the following: $(\alpha, \beta, \rho, Q, \sigma) = (1, 5, 0.5, 100, 6)$, that are suggested to be advantageous.[9] The original AS_{rank} and AS_{elite} were tested as comparative experiments with the same parameter setting as above. σ was set to the number of cities in AS_{elite}. Ten trials were conducted for each algorithm, and each trial consisted of 10000 iterations.

Other studies[6,8] showed the following tendencies concerning the parameters used in conventional ACO: if α is too high compared with β, the algorithm tends to show stagnation behavior without finding good solutions. If α is too low, the algorithm operates like a stochastic multigreedy algorithm. If ρ is close to zero, pheromone evaporates immediately and the algorithm cannot exploit the positive feedback. If ρ is too close to unity, there is the danger of early convergence of the algorithm. Q has rather negligible effects on results. We also observed these tendencies, especially the sensitivities of the parameters α and β, in our preliminary experiments.

Table 1. Computational results on eil51.tsp

Method/r	Best	Avg	Std Dev
AS_{elite}	426 (0.00%)	432.6 (1.55%)	7.07
AS_{rank}	435 (2.11%)	447.7 (4.93%)	11.96
0.01	426 (0.00%)	430.7 (1.10%)	3.47
0.02	428 (0.47%)	430.8 (1.13%)	1.93
0.03	426 (0.00%)	427.2 (0.28%)	1.81
0.04	426 (0.00%)	427.3 (0.31%)	1.25
0.05	426 (0.00%)	426.7 (0.16%)	1.49
0.06	426 (0.00%)	432.8 (1.60%)	6.51
0.07	426 (0.00%)	429.6 (0.85%)	4.95
0.08	426 (0.00%)	440.0 (3.29%)	8.94
0.09	428 (0.47%)	439.1 (3.08%)	8.35
0.10	426 (0.00%)	440.9 (3.50%)	8.35

Method/r, Best, Avg, and Std Dev represent the method or value of random selection rate r, the shortest tour length, the average tour length, and the standard deviation of tour length over 10 trials, respectively. The top two rows show the results of the original AS_{rank} and AS_{elite} methods. The other rows show the results of AS_{rank} with random selection. Values in parentheses are the percentage differences from the optimal tour length (426)

Table 2. Computational results on st70.tsp

Method/r	Best	Avg	Std Dev
AS_{elite}	691 (2.37%)	706.9 (4.73%)	6.61
AS_{rank}	740 (9.63%)	791.7 (17.29%)	26.00
0.01	675 (0.00%)	684.1 (1.35%)	8.14
0.02	677 (0.30%)	682.3 (1.08%)	3.71
0.03	677 (0.30%)	684.4 (1.39%)	7.91
0.04	677 (0.30%)	685.0 (1.48%)	5.56
0.05	676 (0.15%)	683.9 (1.31%)	9.02
0.06	679 (0.59%)	687.1 (1.79%)	5.22
0.07	679 (0.59%)	689.9 (2.21%)	11.17
0.08	681 (0.89%)	692.3 (2.56%)	9.57
0.09	691 (2.37%)	704.7 (4.40%)	11.39
0.10	694 (2.81%)	703.8 (4.27%)	7.64

Table 3. Computational results on kroA100.tsp

Method/r	Best	Avg	Std Dev
AS_{elite}	21361 (0.37%)	21517.7 (1.11%)	86.04
AS_{rank}	21637 (1.67%)	22168.5 (4.17%)	431.32
0.001	21282 (0.00%)	21582.0 (1.41%)	277.04
0.002	21305 (0.11%)	21452.8 (0.80%)	168.82
0.003	21305 (0.11%)	21428.6 (0.69%)	122.76
0.004	21282 (0.00%)	21473.0 (0.90%)	196.23
0.005	21282 (0.00%)	21410.3 (0.60%)	154.81
0.006	21292 (0.05%)	21435.7 (0.72%)	128.74
0.007	21282 (0.00%)	21398.4 (0.55%)	112.05
0.008	21305 (0.11%)	21404.0 (0.57%)	87.14
0.009	21282 (0.00%)	21390.9 (0.51%)	63.68
0.010	21305 (0.11%)	21430.6 (0.70%)	134.97

Our preliminary experiments also showed the following tendency concerning parameter σ for AS_{rank}. If σ is too low, the algorithm tends to show stagnation behavior without finding good solutions. If σ is too high, the ranking mechanism does not work and the behavior becomes like the ant system.

The mechanism of random selection directly controls diversity in the ACO framework, based on our view that diversity control is essential to improve the performance of ACO. Therefore, we aim to improve the performance of ACO by adjusting the random selection rate r even on the premise of rather rough adjustment of the other parameters.

4.2 Comparison of tour length

Table 1 shows the results of the experiments.

Table 1 shows that AS_{elite} outperforms AS_{rank} is this case, similar to the results of the experiments by Stützle and Hoos.[7]

It can be observed that introduction of random selection improves the performance of AS_{rank} on the whole. An optimal solution was found over a wide range of random selection rate and best solutions among three algorithms were obtained when the random selection rate was between 0.03 and 0.06. It is noteworthy that when the random selection rate was 0.05 the result (426.7) outperformed the recently reported result of MMAS + PTS (427.1).[7] In addition, the standard deviation was very small (less than 2) when the random selection rate was between 0.02 and 0.05 compared with the other two algorithms (7.07 and 11.96). This result shows the stability of the extended algorithm.

We also tested the extended algorithm on st70.tsp including 70 cities (its optimal tour length is 675) and kroA100.tsp including 100 cities (its optimal tour length is 21282) from the TSPLIB. The result appeared consistent with the above-mentioned result, in which introduction of random selection reduced the average deviation from the optimal solution (Tables 2 and 3). The optimal solutions were found independently of problem size when the random selection rate was set up appropriately. Also, average tour lengths were greatly improved on all the problems by adopting random selection. Therefore, it seems quite probable that the effectiveness of this mechanism for further larger problems will be shown.

4.3 Diversity of pheromone

The number of different tours found by elite ants and the number of different tours containing an edge or edges by random selection found by elite ants were investigated. This was to clarify how diversity in finding tours, increased by random selection, affect diversity in depositing pheromone (which is correlated with diversity of tours when ρ is small).

Transitions of these two numbers over 1000 iterations are shown in Fig. 2. In the case of AS_{rank} (Fig. 2a), tours found by elite ants converged to one tour without selecting new edges in the early iterations. The reason is considered to be that exploitation was overly accelerated. In the case that the random selection rate was 0.01 (Fig. 2b), tours found by elite ants also converged to nearly one tour, while tours that updated the best tour sometimes contained the edges taken by random selection. However, diversity of pheromone was considered to be not enough in this case, because the number of different tours found by elite ants was small.

Figure 2c shows the case in which the random selection rate was 0.05 and the best performance was achieved. The

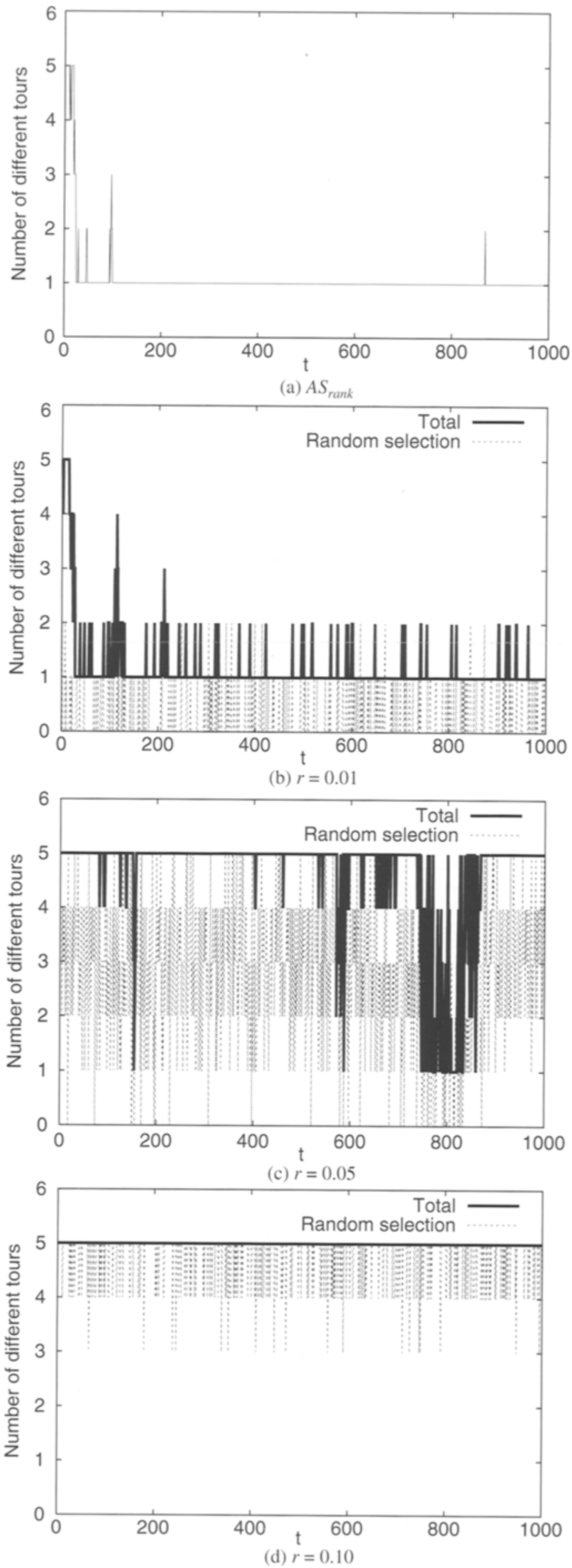

Fig. 2a–d. The number of different tours generated by elite ants and the number of tours including edge(s) selected by random selection generated by elite ants

Table 4. Proportion of tours containing edge(s) selected by random selection

Tour length	All	RS
420–429	22	12 (55%)
430–439	37	13 (35%)
440–449	24	9 (38%)
450–459	24	11 (46%)
460–469	8	4 (50%)
470–479	9	4 (44%)
480–489	9	4 (44%)
490–499	9	4 (44%)
500–589	21	13 (62%)
Total	163	74 (45%)

All expresses the number of times the global best tours were found, RS expresses the number of times those tours found contained edges taken by random selection

number of different tours found by elite ants was nearly 5, which shows that diversity of pheromone was maintained. Approximately half of the tours found by elite ants contained edges taken by random selection.

In the case for which the random selection rate was 0.1 (Fig. 2d), most tours generated by elite ants contained edges taken by random selection. This means that diversity in finding tours increased by random selection directly affected diversity of pheromone. Tours containing edges taken by random selection are worse than those without such edges, on average. Therefore, it can be said that diversity in finding tours overly influenced the positive feedback in ACO.

4.4 Effectiveness of random selection

The effectiveness of random selection, or how strongly the two schemes described in Sect. 3 work, was investigated by focusing on ten trials when the best performance was achieved ($r = 0.05$).

First, in order to test the effectiveness of the first scheme described in Sect. 3, we examined whether global best tours contained the edges taken by random selection when these tours were found. Table 4 shows the number of global best tours containing such edges for every classification of tour length (an interval is 10 when tour length is less than 500). Table 4 shows that the proportion of tours using random selection was more than 30% for every classification of tour length. It should be noted that when optimal or near-optimal tours were found, the proportion was as high as 55%. The average proportion was about 45%, which means that random selection worked well in updating the global best tour without depending on the progress of searching.

In order to test the effectiveness of the second scheme described in Sect. 3, we also examined whether the global best tours consisted of edges on which pheromone was deposited, when new global best tours were found. For this purpose, we discriminated the use of pheromone based on whether each edge had been found by random selection or not, when every pheromone was deposited. Figure 3 shows the average percentage of pheromone deposited by random selection over all edges, when the global best tours were

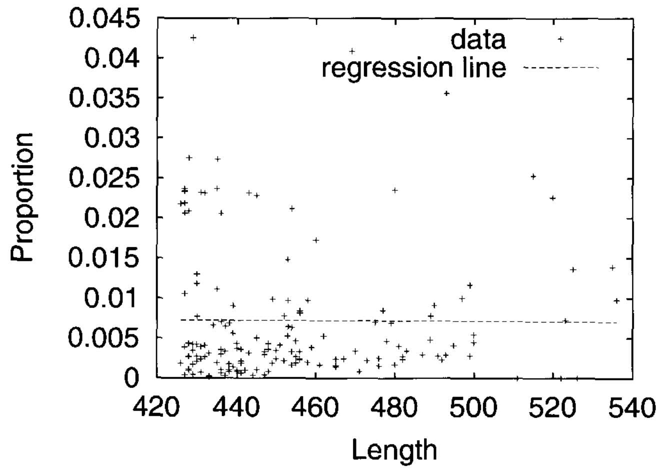

Fig. 3. Proportion of pheromone secreted by random selection

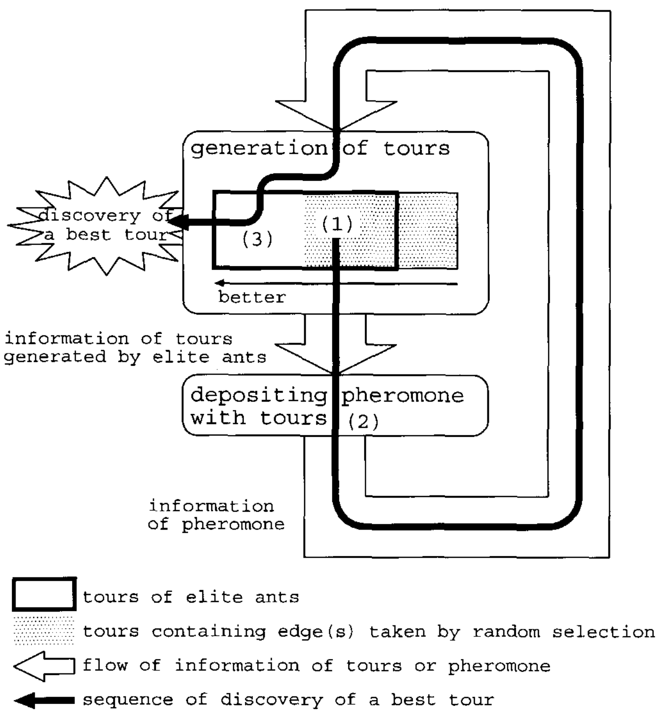

Fig. 4. Conceptual diagram of a feedback mechanism utilizing two types of diversity

found. Each point represents the proportion deposited by random selection and tour length when the global best tour was found. The regression line was drawn by a linear least squares method. It can be seen that pheromone deposited by random selection played an appropriate role without depending on the progress of searching.

This scheme can be summarized as follows: (1) an ant using random selection becomes an elite ant; (2) pheromone is deposited on the tour generated by the ant, which means an increase in pheromone diversity; and (3) another ant in the following generations finds a global best tour by using the pheromone (Fig. 4).

In this scheme, the elite ants are dynamically divided into two classes depending on whether random selection was used or not. It can be said that cooperation between two classes of ants transforms the positive feedback in ACO moderately from the inside with the result that good solutions are found. When the random selection rate is small, it is rare that ants using random selection become elite ants. However, elite ants using random selection appear when the random selection rate becomes larger. When the random selection rate is within a certain range, a mechanism of elitism supports the abovementioned self-regulating mechanism. However this mechanism does not work well and it is difficult to obtain good solutions when the random selection rate is too large.

5 Conclusion

This study investigated the effects of controlling the diversity in the ACO framework. For this purpose, we introduced a mechanism of random selection in AS_{rank} which is one of the newest ACO algorithms. The results have shown that the average deviation from optimal solutions can be reduced by 80%–90%. We have found that the effect of control of diversity in finding tours is adjusted automatically based on adopted mechanisms that realize some sort of elitism. In this scheme, the elite ants are dynamically divided into two classes and cooperation between these two classes of ants transforms the positive feedback in ACO moderately from the inside, which results in finding of good solutions.

We are now investigating the effects of diversity control further in order to address the question: does it work well when larger problems are targeted, when other optimization problems are targeted, or when the mechanism of random selection is introduced in other ACO algorithms?

In addition, this study can be classified as a study of "misperception", because random selection can be regarded as misperception of pheromone information. It has been shown quantitatively that misperception could have a beneficial effect from a collective view point when individuals misperceive incoming information, which leads to an increase in diversity.[10] It would be also interesting to investigate the parallelism between the results by regarding random selection as misperception of pheromone information.

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