Evolution of Cooperation on Different Pairs of Interaction and Replacement Networks with Various Intensity of Selection

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***Abstract*— There are various discussions on the evolution of cooperation on different pairs of interaction network for playing games and the replacement network for imitation of strategies. This paper aims at clarifying the topological relationship between these networks that facilitates the evolution of cooperation by focusing on the intensity of selection for imitation process of strategies. We constructed an agent-based model of the evolutionary prisoner's dilemma on different pairs of interaction and replacement networks. The relationship between these networks can be adjusted by the scale of interaction and reproduction, and the intensity of selection can be adjusted from the almost deterministic selection of the best strategy to the extremely stochastic selection. The evolutionary experiments showed that the larger scale of reproduction than the scale of interaction brought about higher level cooperation when the intensity of selection was high, and the minimum scale of interaction and reproduction was the best for the evolution of cooperation when the intensity of selection was low.**

I. INTRODUCTION

Prisoner's Dilemma has been widely studied for the evolution of cooperation in various scientific fields. A spatial locality has been introduced into evolutionary models as more realistic and ecologically plausible situations of interactions. It is often said that the regular spatial locality facilitates the evolution of cooperation, because cooperators can grow a cluster of their own strategies while defectors cannot [1]–[5]. However, the structures of existing networks of human interactions are more complex than the ones adopted in the studies above, and have several complex properties such as the small-world [6] or the scale-free [7]. Thus, there have been a lot of discussions on the evolution of cooperation on the complex network structures [6], [8]–[11].

There are various discussions on the evolution of cooperation on different pairs of interaction and replacement networks. Each agent plays games against its neighboring agents on a network called an interaction network, and imitates one of the strategies of its neighboring agents on a different network called a replacement network. Under this condition, Ohtsuki et al. discussed the evolution of strategies for the prisoner's dilemma based on a weak selection in which the effects of the payoffs on the fitness difference is relatively small. They showed mathematically that the optimum configuration of the networks for the successful invasion into the sea of defectors by a cooperator was achieved when the average degrees of both networks were the smallest, and the number of the overlapped links was the maximum [12], [13]. Ifti et al. also showed that if the difference between the size (the number of agents) of the interaction and dispersal neighborhood exceeded a certain value, the cooperators could not survive on the spatial and continuous prisoner's dilemma [14]. These studies conclude that both networks should be identical and sparsely connected for the successful evolution of cooperation. A limited dispersal (and interaction) or a population viscosity is known as a mechanism which can bring about such a situation.

On the other hand, it is also said that there is a possibility that a too strong limitation or viscosity rather inhibits the evolution of cooperation. It is mainly due to the fact that they can increase the degree of competitions among relatives in the replacement process as well as (cooperative) interactions among relatives [15]. The several studies have shown that such a situation can occur in the spatial evolution of the strategies for prisoner's dilemma [16]–[19]. For example, Wilson et al. constructed a model of a pure population viscosity in which agents play the prisoner's dilemma games with their neighbors and the best one always deposits its offspring close to itself on a two-dimensional grid [16]. They showed that the evolution of cooperation was less facilitated compared with the control case in which there were interactions among agents with the similar viscosity but there was no local population regulation. Suzuki and Arita also conducted the evolutionary experiments of strategies for the N-person iterated prisoner's dilemma on one-dimensional grid with various settings of the scale of interaction (the number of neighboring members for playing N-person games) and the scale of reproduction (the number of neighboring candidate parents for an offspring in each grid at the next generation) [17]. Experiments showed that higher-level cooperation was achieved when the scale of interaction was relatively small and the scale of reproduction was the largest. This is expected to be due to the fact that the global reproduction allows the successful strategies (cooperators) rapidly and widely occupy the whole population. More recently, Wu and Wang considered two layered graphs of two-dimensional lattices, and analyzed the influence of the difference between the interaction and learning graphs on the evolution of cooperation [19]. They allowed the individuals to learn an adaptive strategy not only from their immediate neighbors but also from their neighbors' neighbors, and so on. They found that the degree of cooperation of the

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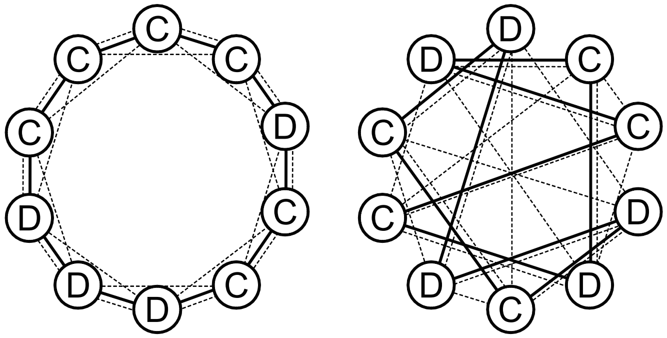


Fig. 1. Examples of the interaction and replacement networks when KI=0 and KR=1, which are generated from a one-dimensional regular network with the degree 2 (left), and from a random network with the average degree 2 (right). Each node has a strategy for the prisoner's dilemma: cooperate (C) or defect (D). The set of links in solid lines represents the interaction network, and that in dotted lines represents the replacement network.

population was increased substantially in a way resembling a coherence-resonance-like behavior when the number of learning edges was increased. Thus, there is still no common agreement on what kind of topological relationship between the interaction and replacement networks can contribute the evolution of cooperation in general.

We focus on the intensity of selection as a key factor that decides the effects of the topological relationships between both network structures on the evolution of cooperation. It is pointed out that the various degrees of the intensity of selection ranging from a weak selection to the strong selection or the so-called imitation dynamics have been adopted so far [20]. Several studies have adopted a pairwise comparison as an imitation mechanism in which each agent chooses another neighbor randomly, and imitates its strategy in stochastically proportional to the difference between its own fitness and the focal neighbor by using the Fermi distribution function. There is a parameter which determines the degree of choosing the best strategy from the two. Wu et al. adopted this type of selection in the models [19] explained above. Vukov et al. and Szabó et al. showed that the optimal condition of the temperature for the maintenance of cooperation depended on the local structures of the graph which represented both interaction and replacement networks [21], [22]. In addition, Traulsen et al. constructed a mathematical model based on a pairwise comparison, and showed that higher intensity of selection in a finite and well-mixed population can reduce the fixation probability of cooperators [20]. However, it is still not clearly understood how the intensity of selection can facilitate the evolution of cooperation on different pairs of interaction and replacement networks.

This paper aims at clarifying what kind of the topological relationship between the interaction and replacement networks can facilitate the evolution of cooperation by focusing on the intensity of selection for imitation process of strategies. For this purpose, we constructed an agent-based model of the evolution of strategies for prisoner's dilemma on different pairs of interaction and replacement networks. In this model, the topological relationship between both networks can be adjusted by the scale of interaction and reproduction, which are the repeat count of adding the links from each individual to its neighbors' neighbors on the initial interaction or reproduction network. Also, the intensity of selection can be adjusted by the temperature parameter based on the Boltzmann distribution selection. By conducting the evolutionary experiments with various settings of the both scales and the intensity of selection, we show that the intensity of selection strongly affects the condition of the scale of interaction and reproduction in which the evolution of cooperation is facilitated.

TABLE I

A PAYOFF MATRIX OF THE PRISONER'S DILEMMA.

opponent player	cooperate	defect
cooperate	$(b-c,\ b-c)$	$(-c,\ b)$
defect	$(b,\ -c)$	(0, 0)

(player's score, opponent's score)
$b > c$

II. MODEL

The N individuals are represented as nodes, and each individual has a strategy for one-shot prisoner's dilemma game (cooperate or defect). The nodes can be connected by two types of links, and they form the interaction network and the replacement network as shown in Fig. 1.

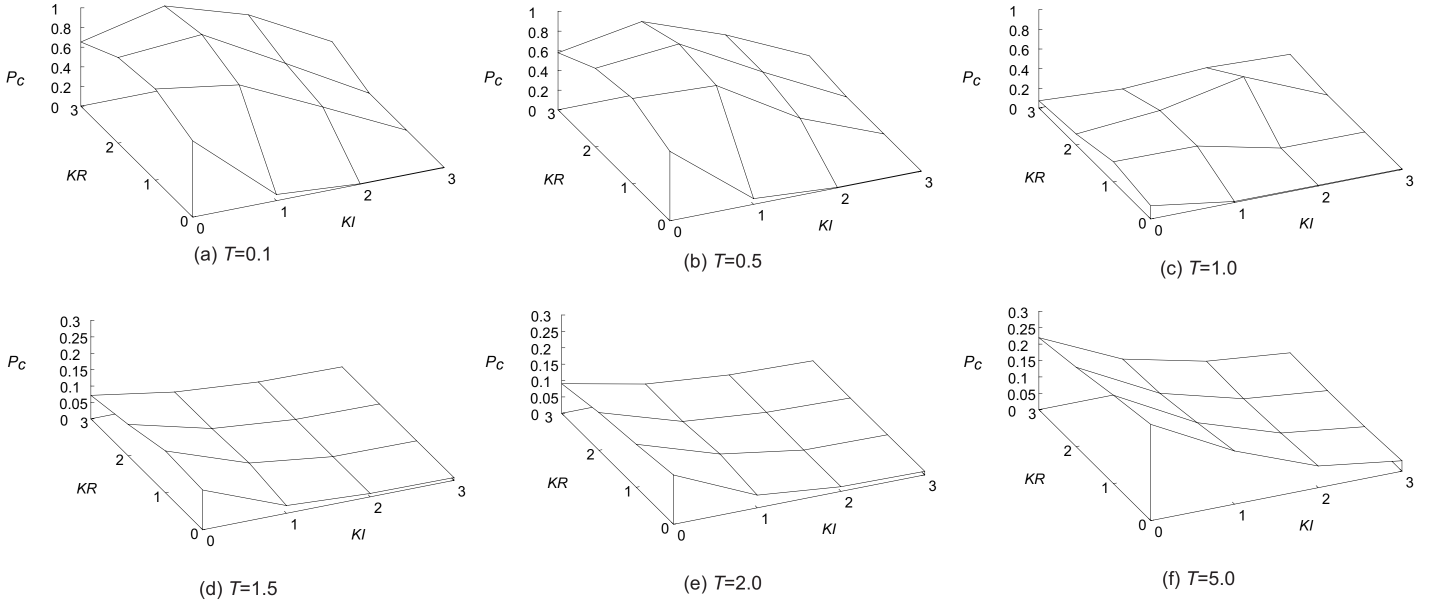
We generate the interaction and replacement networks by using the following procedure:

1) We assume an initial network which determines the initial topology of both interaction and replacement networks. We adopt one-dimensional regular network with the degree 2 or a random network with the average degree 2. The latter is created by the process in which each individual creates a link with a randomly selected other individual.
2) We connect each individual with its all neighbors' neighbors on the interaction (or replacement) network. This is repeated for KI (or KR) times.

The parameters KI and KR determine the topological relationship between the interaction and replacement networks. If $KI \geq KR$, the interaction network includes the replacement network, and vice versa. If KI=KR, both networks are identical.

The game is a two-person version of the prisoner's dilemma of which payoffs are defined in Table I. The parameter b represents the benefit of a cooperative behavior, and c represents its cost. Each step consists of the three phases defined as follows:

1) Each individual plays games against all neighboring individuals on the interaction network. The total payoff is taken as the fitness of each individual.
2) Each individual imitates one of the neighboring individuals' strategies on the replacement network in



(a) T=0.1 (b) T=0.5 (c) T=1.0

(d) T=1.5 (e) T=2.0 (f) T=5.0

Fig. 2. The distribution of the average proportion of cooperation in the case of regular networks.

stochastically proportional to their fitness using the following equation:

$$P_{i,j \in NR_i} = \frac{e^{f_j/T}}{\sum_{k \in NR_i} e^{f_k/T}}, \tag{1}$$

where $P_{i,j}$ is the probability that the individual i imitates the strategy of the individual j, f_j is the fitness of the individual j and NR_i is the set of the neighboring individuals of i on the replacement network. Note that NR_i includes the focal individual i itself. This process is known as the Boltzmann distribution selection which is often used as a decision making mechanism in reinforcement learning. T is the parameter which determines the intensity of selection. If T is sufficiently small, the imitation process becomes almost deterministic and equivalent to the case in which each individual imitates the strategy of the best neighbor. As T becomes larger, the imitation process becomes more stochastic and each individual chooses less adaptive individuals more often. This update process of each strategy occurs at the same time.

3) For each individual who imitated a strategy of other individuals in the phase 2), the inversion of the strategy occurs with a small probability p_m, which corresponds to a misinterpretation of other individuals' strategy[1]

These steps are conducted repeatedly.

TABLE II

THE AVERAGE PROPORTION OF COOPERATORS OVER ALL CONDITIONS OF KI AND KR IN THE CASE OF REGULAR NETWORKS.

T	0.1	0.5	1.0	1.5	2.0	5.0
P_c	0.471	0.419	0.091	0.039	0.047	0.116

III. RESULTS

We conducted evolutionary experiments using the following parameters: $N = 300$ and $p_m = 0.005$, $b = 1.0$ and $c = 0.05$. The initial population was generated on condition that the strategy of each individual was assigned cooperation with a fixed probability p_c= 0.5, otherwise defection.

A. Evolution on Regular Networks

1) General analyses: First, we discuss the results in the case of the evolution on regular networks. In this case, the topology of the interaction or replacement network is the one-dimensional regular network with the degree 2^{KI+1} or 2^{KR+1} respectively.

Fig. 2 shows the average proportion of cooperators (P_c) through the last 200 steps of the trials (500 steps) in various cases of KI and KR (0, $\cdots$, 3) when T= 0.1, 0.5, 1.0, 1.5, 2.0 and 5.0. The x and y axes show the KI and KR, and the z axis is the average proportion of cooperators. Each value is the average over 40 trials. Fig. 2 clearly shows that T strongly affected the condition of KI and KR that brought about the maximum proportion of cooperators in general.

Table II shows the proportion of cooperators averaged over all conditions of KI and KR in Fig. 2. We see that the proportion of cooperators became smaller drastically as

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[1] When there were no mutations of strategies (p_m=0.0), the main results shown in the latter sections did not change except for the increase in the average propotion of cooperators in general. This is expected to be due to the fact that the randomness of the distribution of strategies in the initial population had the similar effect to occurrences of mutations.

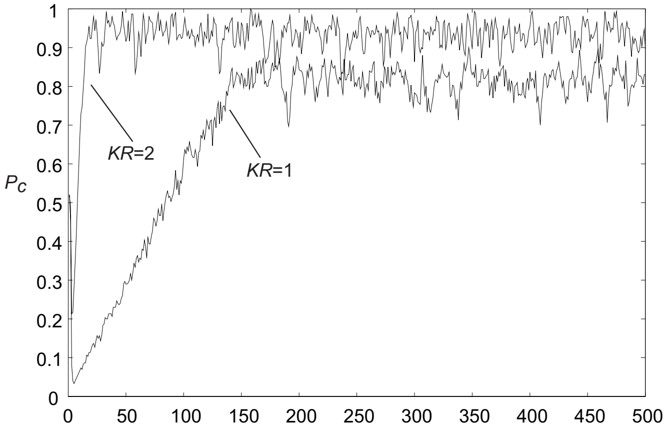

Fig. 3. Example transitons of the proportion of cooperators (P_c) in the cases of KI=1 and KR=1 or 2.

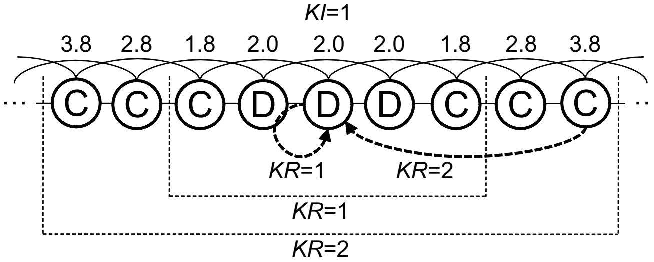

Fig. 4. An example of imitation process when KI=1 and KR=1 or 2. The links represent the interaction network, and each region represents the neighboring individuals for the defector on the center on the replacement network when KR=1 or 2. Each broken arrow also represents a typical imitation process of the focal individual if it imitates the best neighbor.

T increased from 0.1 to 1.5. It means that imitating the better individual's strategy from the neighboring individuals facilitates the evolution of cooperation in these conditions. It is supposed to be due to the two reasons. First, by definition, the fitness of cooperative clusters is larger than those of the clusters of defectors, which enables the individuals to imitate the strategy of cooperators more often when T is small. Second, a stochastic imitation of a strategy makes the distribution of the strategies disordered when T is large. This brings about a situation in which the defectors appear inside the clusters of cooperative individuals by chance, which makes the clusters often collapse.

On the other hand, as T became much larger (2.0 and 5.0), the proportion of cooperators rather tended to become large as shown in Table II. It is due to the fact that the selection process became close to the pure random selection which makes the average proportion of cooperators 0.5.

2) The evolution of strategies with high intensity of selection: In the cases of the small T (0.1 and 0.5), the proportion of cooperators tended to be large when $KR \geq KI$, and tended to be small when $KR < KI$ as shown in Fig. 2 (a) and (b). Note that the behavior of the population when T=0.1 was basically similar to that in the case in which each individual always imitated the best neighbor's strategy except for the result when KI=KR=0 [2] In this case, there was a peak value (0.94) at KI=1 and KR=2, and it decreased as the condition deviated from this optimal one. That is, the evolution of cooperation was strongly facilitated when the scale of reproduction was larger than the scale of interaction if the intensity of selection was sufficiently high. This has been pointed out in several studies [16], [18], [19] as explained before.

Here, we focus on the difference in the behaviors of the population in the cases of "KI=KR=1" and "KI=1 and KR=2" when T=0.1, so as to clarify why the larger scale of reproduction contributed to the evolution of cooperation. Fig. 3 shows the typical transitions of the proportion of cooperators in both cases. When KI=KR=1, after its sudden decrease for several steps from the initial value around 0.5, the proportion of cooperators slowly increased until around the 180th generation and fluctuated around 0.8. On the other hand, in the case of KI=1 and KR=2, the proportion of cooperators rapidly increased until the 30th generation, and fluctuated around 0.95. This difference is due to the difference in the robustness of the cooperative population against invasions by the clusters of mutant defectors. Fig. 4 shows a typical example of such a situation in which there is a cluster of three defectors in the sea of cooperators. Let us assume that each individual always imitates the strategy of the best neighbor. In the former case, the centered defector does not change its own strategy because the fitness of its own and the neighboring defectors are the best (2.0). On the other hand, in the latter case, the centered defector imitates the strategy of the most distant cooperator who has obtained the best fitness (3.8) because it has not been exploited by the focal defector. In both cases, the defectors on the right and left side imitate the strategy of the cooperator whose fitness is 2.8 (KR=1) or 3.8 (KR=2). As a result, the cluster of these defectors rapidly disappears in the next step in the latter case while it can persist and begin to grow again at the subsequent steps in the former case. Such an imitation of the strategy from non-interacting cooperators often occurs when the scale of reproduction is larger than the scale of interaction, and the population is mainly composed of the cooperators.

However, the large scale of reproduction also brings about the two different negative effects on the evolution of cooperation. First, it increases the effect of the invasion by mutant defectors. By definition, a mutant defector appeared in the sea of cooperators is always imitated by the neighboring 2^{KR+1} cooperators at the next step. Second, it decreases the intensity of selection implicitly because the individuals must compete for being imitated against other $2^{KR+1}+1$ individuals, which can decrease the best neighbor's probability of being imitated. Thus, the net benefit of the large scale of reproduction depends on the balances between these factors, and they were supposed to be balanced when KI=1 and

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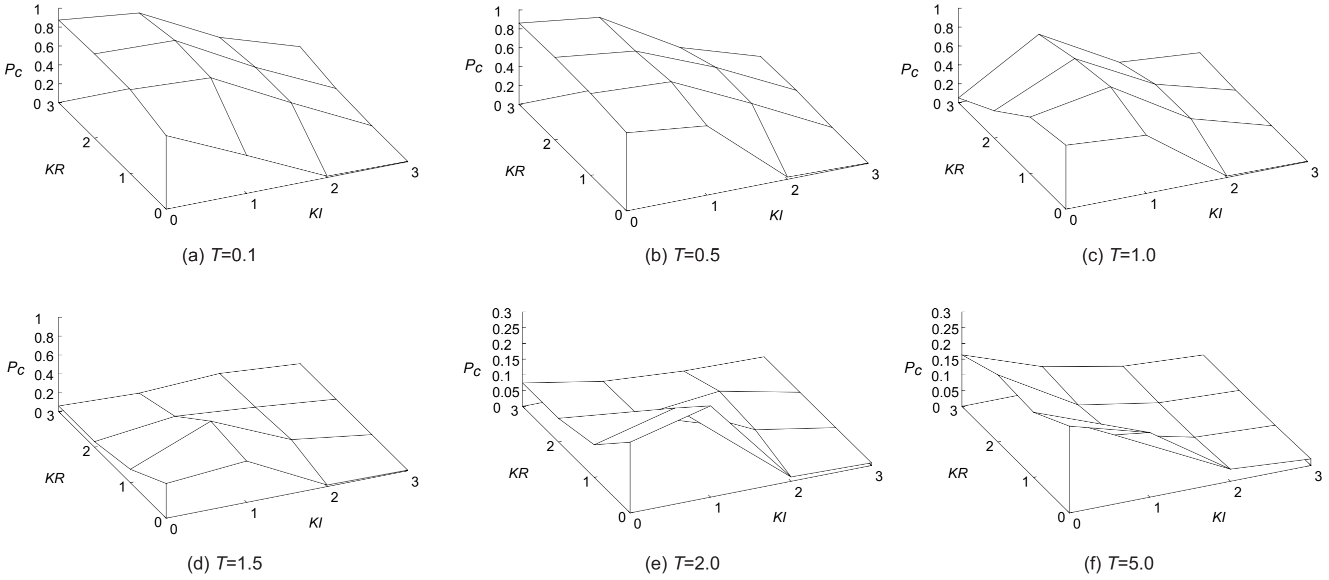
[2] In this exceptional condition, the proportion of cooperators was almost zero because the cooperative clusters could not grow due to the strongest and deterministic restriction of the imitation process.



(a) T=0.1 (b) T=0.5 (c) T=1.0

(d) T=1.5 (e) T=2.0 (f) T=5.0

Fig. 5. The average proportion of cooperation in the case of random networks.

TABLE III

THE AVERAGE PROPORTION OF COOPERATORS OVER ALL CONDITIONS OF KI AND KR IN THE CASE OF RANDOM NETWORKS.

T	0.1	0.5	1.0	1.5	2.0	5.0
P_c	0.477	0.482	0.303	0.129	0.072	0.092

KR=2. Because lower intensity reduces the former benefit, the average proportion of cooperators tended to become smaller especially when $KR \geq KI$ as T increased from 0.1 to 1.0. As a result, when T=1.0, it finally was the maximum when KI and KR were identical (KI=KR=2).

It is clear that the cooperators could not occupy the population when $KR < KI$. In these cases, each individual plays games against all candidate individuals for imitation, and thus the evolutionary process becomes similar to that of the well-mixed population as the difference between KI and KR becomes large.

3) The evolution of strategies with low intensity of selection: In the cases of the large T (1.5, 2.0 and 5.0), the proportion of cooperators was relatively large when KI was small. Especially, in the cases of T=2.0 and 5.0, a peak value (0.15 and 0.29) appeared when the scale of interaction and reproduction were the smallest ($KI = KR = 0$). In these conditions, the proportion of cooperators is quite small due to the reasons explained above. Thus, the strongly limited interactions and reproduction with neighboring individuals is expected to be the best for the cooperative clusters to survive in the sea of defectors so as not to being exploited by the non-conspecifics (defectors).

B. Evolution on Random Networks

Fig. 5 shows the results in the case of the evolution on the random network with the average degree 2[3]. It shows that the general effects of KI, KR and T on the evolution of cooperation were basically similar to those in the case of regular networks. If T was the smallest (0.1), the proportion of cooperators was the largest (0.89) when the scale of reproduction was larger than the scale of interaction ($KI = 0$ and $KR = 2$). If T was the largest (5.0), the proportion of cooperators was the largest (0.28) when both scales of interaction and reproduction were the smallest ($KI = 0$ and $KR = 0$).

Table III shows the proportion of cooperators averaged over all conditions of KI and KR in Fig. 5. We see that it tended to be larger than that in the previous cases when $T \leq 2.0$. In the random networks, there is a variance in the degree between individuals while there is no difference in the regular networks. This is supposed to be one of the reasons for such a higher level cooperation because the cooperators with the larger number of links can exist stably by forming a hub of cooperative individuals. We also see that the proportion of cooperators was the highest when KI=1 and KR=0 if T was intermediate (1.5 and 2.0), which is different from the previous result. It is supposed to be due to the fact that increasing the scale of interaction amplifies the variance in the degree between individuals.

As a whole, we can say that the intensity of selection strongly affects the condition of the scale of interaction and reproduction in which the evolution of cooperation is

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[3] We also conducted the experiments on condition that the average degree of the initial random network was 4. The main results did not change but we found that increasing the average degree had the similar effect to decreasing the intensity of selection, which is expected to be due to the increase in the number of competing individuals during reproduction processes.

facilitated. The larger scale of reproduction compared with the scale of interaction brings about higher level cooperation when the intensity of selection is large, and the minimum scale of interaction and reproduction is essential for the evolution of cooperation when the intensity of selection is small.

IV. CONCLUSION

We have discussed the evolutionary dynamics of cooperative behaviors on the different pairs of interaction and replacement networks under the various intensity of selection. We constructed an agent-based model of the evolution of strategies for prisoner's dilemma in which the scale of interaction and reproduction can be adjusted independently and the intensity of selection can also be adjusted from the almost deterministic selection of the best strategy to the extremely stochastic selection which corresponds to the weak selection.

By conducting the experiments with the various conditions of the two networks, we found that the intensity of selection strongly affects the condition of the scale of interaction and reproduction in which the evolution of cooperation is strongly facilitated. When the intensity of selection was sufficiently high, the evolution of cooperation was strongly facilitated when the scale of reproduction was slightly larger than the scale of interaction. It is due to the fact that the larger scale of reproduction brings about the robustness of the cooperative population against invasion by the clusters of mutant defectors. On the other hand, when the intensity of selection was sufficiently low, the evolution of cooperation was facilitated when both scales were the smallest, because the strongly limited interaction and reproduction was the best for the cooperators to manage to survive in the sea of the defectors. These results were observed in both one-dimensional regular networks and random networks.

It also should be noticed that the behavior of the population was varied continuously between these two extreme cases by altering the intensity of selection gradually. The former case seems consistent with Wu et al.'s result because they showed the similar results under the condition which is supposed to correspond to the relatively high intensity of selection [19], and the latter seems consistent with Ohtsuki et al.'s because they derived mathematically the similar optimal condition for the evolution of cooperation under the assumption of the weak selection [12], [13]. Our finding implies that the reason for these different conclusions can be explained at least in part by the difference in the intensity of selection adopted in these models. On the other hand, we also found that the obtained results are not consistent with the results by Ifti et al.'s [14] and the results of our previous model [17]. This might be due to the fact that they assumed the different settings of games such as the continuous prisoner's dilemma game in the former model or the iterated N-person prisoner's dilemma games and the use of the averaged payoff as the fitness in the latter model.

Future work includes the analyses of the behaviors of the population on the different topology of the networks.

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